

Math 101 Homework #4

This is due on Wednesday, April 15, at the beginning of class. It is worth 25 points. Late papers lose 3 points per day or partial day late. You may work by yourself or in groups of two, but there should be no communication between groups.

This assignment develops the “Stable Marriage Problem” which represents a different type of “voting” than we have looked at so far. The problem consists of a “courtship” phase followed by an “affair” phase.

Answer all of the questions asked below and write a report on the problem. The report should start with an introduction, flow nicely from problem to problem, and finish with a conclusion. As you can see in the grading criteria on the back, your grade depends on doing the math correctly and communicating it well in writing.

The dating pool consists of six men (Adam, Bob, Chuck, Dan, Ed and Fred) and six women (Amy, Betsy, Cate, Debbie, Erin and Fran). Each person rates the members of the opposite sex in order of whom they would like to marry. The ratings are:

Adam: Amy,Cate,Debbie,Betsy,Fran,Erin	Amy: Bob,Adam,Chuck,Fred,Ed,Dan
Bob: Amy,Betsy,Debbie,Cate,Erin,Fran	Betsy: Adam,Chuck,Bob,Ed,Dan,Fred
Chuck: Amy,Cate,Debbie,Erin,Betsy,Fran	Cate: Dan,Bob,Chuck,Adam,Ed,Fred
Dan: Betsy,Cate,Amy,Erin,Fran,Debbie	Debbie: Adam,Chuck,Dan,Ed,Fred,Bob
Ed: Amy,Debbie,Erin,Fran,Cate,Betsy	Erin: Ed,Dan,Adam,Fred,Bob,Chuck
Fred: Debbie,Betsy,Amy,Fran,Erin,Cate	Fran: Chuck,Fred,Dan,Bob,Adam,Ed

In the first round of courtship, each man “proposes” to his top choice. For each woman who receives a proposal, she provisionally accepts the proposal from her highest-rated male and rejects proposals from lower-rated males.

In the second round, each man who was rejected in round one proposes to his next-highest-rated woman. As before, each woman with proposals (from this round or left over from the previous round) provisionally accepts the best proposal and rejects all others.

This process continues until all females have exactly one proposal on the table. At this point, the process stops and the six couples get married.

Who does each person end up marrying?

Why is Betsy not as happy as Amy? Why is Fred not as happy as Bob?

The next stage is where the “unhappy” people try to have an affair with somebody who is higher on their list than their spouse. We assume that affairs are permanent in that all affairs are discovered and the previous marriages break up. We also assume that each person keeps the same preference list as before.

Which women would Fred like to have an affair with? Why will he not be able to get anybody to agree to it? Explain why (under these rules) no man will ever have an affair. Of course, this also means that no woman will have an affair, either. In this sense, the six marriages are stable.

Why would marriages Adam-Amy and Bob-Betsy be unstable?

This courtship scheme is guaranteed to produce stable marriages (as defined above), but there are other stable marriage pairs (Bob-Amy, Chuck-Betsy, Dan-Cate, Adam-Debbie, Erin-Ed, Fred-Fran is another one). This raises the question as to what type of stable marriage our courtship scheme produced. In particular, is it better for the men or for the women? One way to answer this is to ask if anybody could have done better. Could Fred have done better? If he married somebody higher on his preference list than his wife, could it have been a stable marriage? Could Betsy have done better? Is it possible for her to marry somebody higher on her preference list and have it be a stable marriage?

Here is a helpful hint for answering the previous questions. (You still need to explain why your answer makes sense, but at least this will tell you the right answer.) Among all possible stable marriage combinations, the one that we used gives all men their best possible mate and gives all women their worst possible mate. Given this, are you better off being the one doing the proposing or the one that gets to say yes or no?

Like all mathematical models of real situations, this one is oversimplified. What are some realistic and unrealistic aspects of our courtship and affair rules? Do you think that this model tells us anything about marriage relationships?

Grading Criteria

- A:** good introduction, all questions are answered, all answers are correct, explanations are clear, there is a good flow to the writing, all of the work is attached and is readable
- B:** one of the above elements is missing. For example, there is no introduction or one of the questions is not answered. All of the other elements of the A paper are present.
- C:** two or three questions/elements of the A paper are missing (e.g., 2 questions are unanswered). The report is hard to understand in places because of poor explanations or important aspects are missing.
- D:** several questions are unanswered or the writing is disorganized and the explanations are minimal. Even though most questions are answered correctly, it appears that the assignment was completed in the least amount of time possible.
- F:** several questions are unanswered or answered incorrectly, the writing is disorganized and the explanations are minimal.