

Problem Solving Seminar (9/28/09)
Modular Arithmetic

1. Suppose we have a set of integers with the property that the average of any three of them is not an integer. Determine the maximum number of elements the set can have.
2. Compute $5^{2009} \pmod{7}$
3. A tropical island contains R chameleons, G chameleons and B chameleons. When two chameleons of different color meet, they change to the third color. For what triples (R, G, B) can all the chameleons eventually be the same color?
4. Can a positive integer each of whose digits are 6 or 0 be a perfect square?
5. Prove that 19 does not divide $4n^2 + 4$ for any integer n .
6. Find the longest sequence of equal non-zero digits in which the square of an integer can terminate and find the smallest integer that terminates in the sequence. (Putnam 1970)
7. For a triple of integers of the form $\{n, n + 4, n + 8\}$ is it possible that all three are prime?
8. Assume that p is prime. Prove that $x^2 = 1 \pmod{p}$ iff $x = \pm 1 \pmod{p}$.