

Problem Solving Seminar (9/8)

(These problems are from *The Inquisitive Problem Solver* by Vaderlind, Guy and Larson.)

1. You have 43 pieces of fencing whose lengths are 1, 2, 3, ..., 43 meters. Can you build a square fence using all of these pieces (without any bending)? (Follow-up question)
2. The percentage of math majors in a class is more than 60% but less than 65%. What is the smallest number of people who could be in the class?
3. A sports team has a winning percentage less than 75%. After winning a number of games, the winning percentage is greater than 75%. Was there a time at which the team's winning percentage was exactly 75%?
4. Nine pieces of paper are numbered 1 to 9 and placed face up on a table. Two players take turns choosing pieces of paper until one player wins by having numbers that add up to 15. Should the first player always win?
5. Let A, B, C, D, E, F, G, H, I be any nine positive integers and let a, b, c, d, e, f, g, h, i be any permutation (re-arrangement) of those numbers. Can the permutation be chosen so that the product

$$(A - a)(B - b)(C - c)(D - d)(E - e)(F - f)(G - g)(H - h)(I - i)$$

is an odd number? (Follow-up question)

6. In a group of 20 students, each student is required to buy lunch for 8 other students. Is it always going to happen that there are two students who buy each other's lunch? (Follow-up question)
7. I have chosen five positive integers A, B, C, D, E. If you give me numbers v, w, x, y, z then I will give you the dot product $vA+wB+xC+yD+zE$. You can do this until you know my original numbers. What is the fewest number of calculations needed to determine my number?