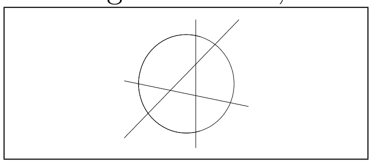
Problem Solving Seminar (9/15)

(These problems are from *The Inquisitive Problem Solver* by Vaderlind, Guy and Larson.)

1. Three lines divide a circle into seven regions. Can you arrange the numbers 1 to 7 in these regions so that, for each line, the sums of the numbers on either side are the same?



2. You have two sandglasses. One measures a 9-minute interval and the other a 13-minute interval. Starting with both sandglasses empty on one side, determine how to measure a 30-minute interval. Which time intervals can be measured using these sandglasses?

3. In a round-robin tournament, medals are awarded as follows. Player A earns a medal if for each other player B, either A beat B or A beat a player who beat B. Is it possible that no medals will be awarded? Is it possible that everybody will be awarded a medal?

4. In the round-robin tournament of problem 3, suppose that two players have the same number of wins. Is it then necessarily true that there exist three players A, B and C such that A beat B, B beats C and C beats A?

5. The number 42 can be written in three different ways as the sum of two or more consecutive positive integers:

$$42 = 13 + 14 + 15 = 9 + 10 + 11 + 12 = 3 + 4 + 5 + 6 + 7 + 8 + 9$$

   Can you write 105 in eight different ways as the sum of two or more consecutive positive integers? Does your answer change if the numbers don't all have to be positive?

6. Find the last digit of each expression: (a) $4^{4^{4^{4^4}}}$ (b) $5^{5^{5^{5^5}}}$ (c) $6^{6^{6^{6^6}}}$ (d) $7^{7^{7^{7^7}}}$ (e) $8^{8^{8^{8^8}}}$

7. Tic-Tac-Toe, played on a 3x3 grid, is not winnable if both players play correctly. Take the 3x3 grid and add a tenth square in the upper right. Is this game winnable?

8. Two people start counting at the same time and at the same speed. Person A starts at 70 and goes forward by threes (70, 73, ...) while person B starts at 2006 and comes down by sevens (2006, 1999, ...). Will there ever be a time at which the two numbers counted are less than 4 apart?

9. For what values of n can the set $\{1, 2, 3, ..., n\}$ be partitioned into two or more subsets such that the products of all members of the subsets are equal? (Hint: try $n = 25$.)

10. A group of 13 coins are identically stamped except for a unique ID number. Twelve coins are counterfeit and one is gold. The gold coin has a different weight. An assistant is available with a pair of scales and a box of weights. You may give the assistant instructions for three separate weighings, which will be done in a separate room. The assistant will tell you only how much weight was needed to balance the scale on each weighing. Determine a strategy that will identify the gold coin.