

Problem Solving Seminar (9/22)

(These problems are from the 2003 and 2004 AIME and the 2005 AMC-8.)

1. Compute $1,000,000.000001^2 - 999,999.999999^2$.
2. For how many integers n is $\log_7 n^2 < 3$?
3. If $\frac{a}{b} = \frac{c}{d} = \frac{5}{8}$, compute $\frac{a^2 + 2006c^2}{b^2 + 2006d^2}$.
4. How many three-digit numbers are divisible by 13?
5. Write $\frac{((3!)!)!}{3!}$ as $k \cdot n!$ where k and n are integers and n is as large as possible.
6. 100 concentric circles have radii 1, 2, 3, ..., 100. The inner circle is colored red, and the gaps between circles are colored green and red alternately. Find the ratio of the red areas to the green areas.
7. In an increasing sequence of 4 positive integers, the first three form an arithmetic sequence, the last three form a geometric sequence and the first and last numbers differ by 30. Find the numbers.
8. If $\log_{10} \sin x + \log_{10} \cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$ find n .
9. Let A be a set of m consecutive integers whose sum is $2m$. Let B be a set of $2m$ consecutive integers whose sum is m . If the absolute value of the difference between the largest elements of each set is 99, find m .
10. Find the coefficient of x^2 in the expansion of $(1-x)(1+2x)(1-3x) \cdots (1+14x)(1-15x)$.