Problem Solving Seminar (9/29)

(These problems are from the 2001 and 2002 AMC-12.)

1. Write the product of all the positive odd integers less than 10,000 as $\frac{10,000!}{2^k n!}$ for integers k and n.
2. Let f be a piecewise linear function whose graph (reading left to right) increases through $(-7,-4)$ to a relative max at $(-2,6)$, decreases to a relative min at $(0,0)$, increases to a relative max at $(1,6)$, then decreases through $(5,-6)$. How many solutions are there to the equation $f(f(x)) = 6$?
3. Define $P(n)$ to be the product of the digits of n, so that for example $P(23) = 6$. Define $S(n)$ to be the sum of the digits of n, so that for example $S(23) = 5$. If N is a 2-digit number such that $N = P(N) + S(N)$, what is the units digit of N?
4. Let a and b both be 1-digit non-negative integers, not both 9 and not both 0. If the repeated decimal $0.ababab\cdots$ is expressed in simplest terms, what are the possible denominators?
5. By definition, $2^{2^{2^2}} = 2^{2^4} = 2^{16} = 65,536$. Using different orders of operations, what other values would be possible?
6. Find all real ordered pairs (a, b) such that $(a + bi)^{2002} = a - bi$.
7. Let P be a fourth-order polynomial with leading coefficient 1, integer coefficients and exactly 2 real zeros, both of which are integers. Find all possible complex zeros of the form $\frac{1}{2} \pm bi$.
8. A sequence starts with $a_1 = x$, $a_2 = 2000$ and $a_n = a_{n-1}a_{n+1} - 1$. Find all values of x such that 2001 is in the sequence.