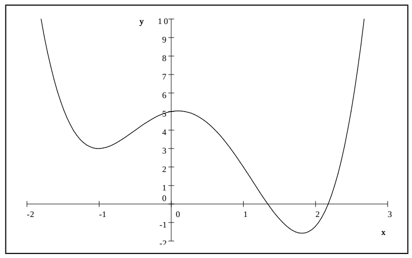
Problem Solving Seminar (10/6)

(These problems are from the 2000 and 2002 AMC-12.)

1. The graph shows a portion of the graph of $P(x) = x^4 + ax^3 + bx^2 + cx + d$ for constants a, b, c and d. List the following in ascending order: $A = P(-1)$, $B =$ the product of the zeros of P, $C =$ the product of the non-real zeros of P, $D =$ the sum of the coefficients of P, $E =$ the sum of the real zeros of P.



2. In a lottery, you pick 6 different integers from 1 to 46 inclusive. How many different ways can this be done such that the sum of the base-10 logarithms of the numbers is an integer?

3. Find the product xyz given that $x + \frac{1}{y} = 4$, $y + \frac{1}{z} = 1$ and $z + \frac{1}{x} = \frac{7}{3}$.

4. Day 300 of year N is a Tuesday. So is day 200 of year $N + 1$. What is day 100 of year $N - 1$?

5. Find all x such that the mean, median and mode of $\{10, 2, 5, 2, 4, 2, x\}$ form a non-constant arithmetic sequence.

6. A unit circle is surrounded by 6 unit circles placed tangent to each other, then a large circle is circumscribed around the ring of circles. Find the area inside the large circle that is not inside of one of the unit circles.

7. Let P be a fourth-order polynomial with leading coefficient 1, integer coefficients and exactly 2 real zeros, both of which are integers. Find all possible complex zeros of the form $\frac{1}{2} \pm bi$ where b is a rational number.

8. A sequence starts with $a_1 = x$, $a_2 = 2000$ and $a_n = a_{n-1}a_{n+1} - 1$. Find all values of x such that 2001 is in the sequence.