Problem Solving Seminar (11/10)

(These problems are from *The Inquisitive Problem Solver*)

1. 64 players take part in a single-elimination tennis tournament. The number of matches that ended 3-0 was twice the number of matches that ended 3-1, which was twice the number of matches that ended 3-2. How many sets were played?

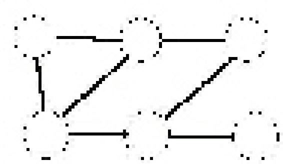
2. Place a $+1$ or a -1 in each square of a 7-by-11 grid. For each row and each column, compute the product of its elements. Is it possible to choose the $+1$'s and -1's so that the 18 products add up to 0?

3. Place the numbers 1 to 16 in a 4-by-4 grid such that you can go in order from 1 to 16 by moving along adjacent squares (diagonals don't count as adjacent). Can this be done with the numbers placed as given?

	11	10	
	7		

4. Place 1 to 16 in a blank grid so that 16 is adjacent to 1.

5. Write $a \to b$ if the circles connected to a have numbers that add to b. Place the numbers 0,1,2,3,4,5 in the circles in such a way that $0 \to 4$, $1 \to 12$, $2 \to 7$, $3 \to 8$, $4 \to 3$ and $5 \to 4$.



6. Beth has a hiding place that starts with 35 coins, 38 books and 39 pieces of candy. Her sister Kelly "borrows" items in the following way: she takes 1 each of two different items and adds in 1 of the third item. If it gets to the point where all items in the hiding place are of the same kind, which is it? How many items of that kind are possible?

7. Is it possible to find positive integers A and B such that $A^2 - B^2 = 631$? how about $A^3 - B^3 = 631$? $A^4 - B^4 = 631$?

8. A game is played on a 9-by-10 rectangular board. Players take turns placing x's in 1 to 6 adjacent squares of a row or column. The winner is the person who places an x in the last empty square. Is there a winning strategy? What if the board is 13-by-13?

9. Is 600! divisible by 7^{98}?

10. With "rectangular" packing, $2n$ balls of diameter 1 can be packed in a 1-by-2-by-n box. Show how to pack more balls if $n \geq 238$.