Math 321: Vector Calculus **2022**

Calculus, Smith & Minton 4th edition, Chapters 10-14

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office hours by appointment at calendly.com/minton/15min

Course Objectives: *Continue to learn how to do mathematics!* Mathematics is a problem-solving discipline, and we all have room to improve. To develop as problem-solvers, we must focus on technique and not on memorization. In this course, we develop an understanding of the theory and elementary applications of vectors, vector-valued functions, multiple integration, line integrals, and the integral theorems of vector calculus. Living in a 3-spatial-dimensional world, it is clearly necessary to use 3-dimensional functions if we are to realistically model the world. Unfortunately, the graphical cues that are so helpful for functions of one variable are not as easy to visualize in 3 or more dimensions. The calculations are sometimes more difficult, requiring us to extend our notion of integration. The different types of integrals are beautifully connected at the end of the course in a series of fundamental theorems.

Intended Learning Outcomes: At the end of the course, successful students will be able to

- Apply techniques of differentiation and integration to solve problems involving functions of two or more variables and vector functions
- Understand the role of vector calculus in modern mathematics
- Determine when different coordinate systems are appropriate
- Distinguish among various types of integrals, and determine when to use each type

Attendance Policy: Attendance and attention in class is essential! Work hard in class and you will have little need for studying the night before a test. You are responsible for everything done in class, through attendance and sharing class notes with classmates. If you miss a class, e-mail or call me before class is over and explain why. With the first unexplained absence, you and your advisor and the registrar will be warned that another unexplained absence will result in removal from the course. **If you have two unexplained absences, you will be dropped from the course.**

Equipment: We will use Mathematica in class, on homework assignments and on tests. You are encouraged to get a copy installed on a laptop or desktop. This is free! Take advantage of this great offer. We will practice using Mathematica in class. There will be homework problems and test questions that will be very hard to work without Mathematica! Please note that Mathematica is used extensively in other upper-level mathematics courses.

Study Problems: In each section of the book that we cover, I will give you a list of problems to study. Work as many of these problems as you can! There are some topics in the book that you are not responsible for learning; the study problems indicate what topics you will see on tests.

Worksheets and Reports: Eight problems will be given to you as worksheets to work on in class. Group work is encouraged. Complete each worksheet and turn in the next class for a grade. For one worksheet problem each half of the course, **individually** write a report of the problem, with an introduction, any details of calculations that are important, and a conclusion. The first report is due on Wednesday, 10/26 and the second report is due on Monday, 12/12. You may ask me for help in the form of specific questions, not proofreading. Extra credit can be earned by exploring the problem farther than was done in the worksheet. Have fun!

Co-Curricular: During the course of the semester, you must attend at least three approved co-curricular events offered by the MCSP department. For each, write a two-paragraph reflection paper, giving a brief summary of the talk and expanding on some aspect of particular interest to you. Reports are due within two weeks of the talk. One report must be turned in by 10/14.

I expect you to spend at least 12 hours of work each week inside and outside of class.

Academic Integrity: The college policy is fully supported. Reports are to be your own work though you may ask me for help. No other outside sources are allowed. Tests are closed notes, closed book, no electronic devices allowed. Anything that requires Mathematica will be given as take-home work. For that, no sources other than Mathematica files posted on Inquire are allowed.

Tests: Mastery-based testing on the topics on the next page. Test questions are graded on a mastery/no mastery basis. You will have multiple chances to demonstrate mastery of a topic. Once you do so, you will not be further tested on that topic. So, if you do not achieve mastery of a topic on one test, new problems on that topic will be given on subsequent tests. My judgment of mastery will be based on whether you show that you fully understand the question; your arithmetic and algebra do not have to be perfect, but there should be no flaws in your approach to the problem. In most cases, a topic will have multiple parts. If you get part (a) right but not part (b), you will get credit for ½ mastery. When you retake the topic, you must work all parts of the topic. Your grade will not be lowered if you do not master as many parts. While Mastery grading has the downside of no partial credit, the upsides include the ability to completely make up for early deficiencies. You always have the possibility of demonstrating mastery on 100% of the topics for the semester. The intent is not to allow you to delay learning topics to the end of the course, but to encourage you to fully understand the early topics so that later topics can make sense. The exam is Friday, December 16, 8:30-11:30 and will consist of all 19 content areas.

Lab Assignment: The day before fall break will be spent working on several aspects of projectile motion. You will be given several questions, some of which require calculation and some of which require Mathematica. Answer all of the questions and write it up as a nice report. It will be graded as a Mastery topic. If your first submission is not acceptable, you will have a chance to revise it, up to once per week, until it passes mastery.

Grading:
Tests and Exam: 55%
Worksheets: 24%
Reports: 16%
Class participation and co-curricular: 5%
Grades may be curved up based on extenuating circumstances, including improvement as the semester goes on.

A: 93-100
A-: 90-92
B+: 87-89
B : 83-86
B-: 80-82
C+: 77-79
C: 73-76
C-: 70-72
D+: 67-69
D: 63-67
D-: 60-62
F: 59 and below

Mastery Topics

1. Dot Product
2. Cross Product
3. Lines and Planes
4. Surfaces in Space
5. Vector Functions
6. Motion in Space
7. Parametric Surfaces
8. Double Integrals
9. Triple Integrals
10. Spherical Coordinates
11. Vector Fields
12. Line Integrals
13. Independence of Path
14. Green's Theorem
15. Curl and Divergence
16. Surface Integrals
17. Divergence Theorem
18. Stokes Theorem
19. Applications
20. Lab Report

Test/Exam score = $4x + 20$ where x is the number of topics mastered.

The Writing Center @ Roanoke College, located on the Lower Level of Fintel Library (Room 15), offers free tutorials focused on writing projects and oral presentations for students working in any field. Writers and presenters at all levels of competence may visit the Writing Center at any point in their process—including brainstorming, drafting, organizing, editing, or polishing presentation skills—to talk with trained peer tutors in informal, one-on-one sessions. The Writing Center is open Sunday through Thursday from 4 to 9 PM. Simply stop in, or schedule an appointment at www.roanoke.edu/writingcenter. Questions? Email writingcenter@roanoke.edu or call 540-375-4949.

Accessible Education Services (AES) is located in the Goode-Pasfield Center for Learning and Teaching in **Fintel Library**. AES provides reasonable accommodations to students with documented disabilities. To register for services, students must self-identify to AES, complete the registration process, and provide current documentation of a disability along with recommendations from the qualified specialist. Please contact Becky Harman, Assistant Director of Academic Services for Accessible Education, at 540-375-2247 or by e-mail at aes@roanoke.edu to schedule an appointment. If you have registered with AES in the past and would like to receive academic accommodations for this semester, please contact Becky Harman at your earliest convenience to schedule an appointment and/or obtain your accommodation letter for the current semester.

Subject Tutoring, located on the lower level of Fintel Library (Room 5), is open 4-9 PM, Sunday-Thursday. Subject Tutors are highly trained, current students who offer free, one-on-one (and small group) tutorials in over 80 courses taught at Roanoke College, including: Business, Economics, Mathematics, INQ 240, Modern Languages, Lab Sciences, and Social Sciences. Check out all available subjects and schedule 30- or 60-minute appointments at www.roanoke.edu/tutoring. If you have a question, feel free to stop by, or contact us at subject_tutoring@roanoke.edu or 540-375-2590. See you soon!

Student Health & Counseling Services supports students through in-person health appointments, in-person counseling, 24/7 telehealth (TimelyCare), Therapy Assistance Online, as well as resources related to general wellness, LGBTQ+, sexual assault, substance abuse, and suicide prevention. Unmet health needs can negatively impact your performance in this course. Student Health & Counseling Services can help. Please see https://www.roanoke.edu/shcs for more information and to access services.

Date	Sections	Topics	Study Problems
W 8/31	9.2	Parametric Equations	
F 9/2	10.1-2	Vector Basics	p.703 15-24 p.711 11-18,33-40
M 9/5	10.3-4	Dot, Cross Product	p.719 3-14,19-22,31-34 p.731 5-8,11-14,35-38
W 9/7	10.3-4	continued	worksheet #1
F 9/9	10.5	Lines and Planes, M1-2	p.740 1-8,15-24
M 9/12	10.6	Surfaces in Space	p.751 1-8,17-22,29-38
W 9/14	11.1	Vector Functions	p.764 1-6,11-16,27 worksheet #2
F 9/16	11.3	Motion in Space, M3-4	p.785 5-8,23-26,51-54
M 9/19	11.6	Parametric Surfaces	p.810 1-5,13-16(M),21-24,27
W 9/21	11.6	continued	worksheet #3
F 9/23	13.1	Double Integrals, M5-6	p.922 11-14,25-30,37-40
M 9/26	13.3	Double Integrals	p.939 7-10,17-28,31-36
W 9/28	13.3	continued	
F 9/30	Mastery testing 1-8		
M 10/3	13.5	Triple Integrals	p.954 3-10,23-28
W 10/5	13.6	Cylindrical	p.962 9-16,25-32,41-44 worksheet #4
F 10/7	13.7	Spherical	p.969 15-24,27-32,49-52
M 10/10	13.7	continued	
W 10/12	Mastery testing 1-10		
F 10/14	Lab Day		
Fall Break 10/17-21			
M 10/24	14.1	Vector Fields	p.995 1-4,9-12, 19-22,29-36,41-46
W 10/26	14.1	Vector Fields	worksheet #5
F 10/28	14.2	Line Integrals	p.1010 5-12,17-26
M 10/31	14.2	Line Integrals	p.1010 31-42
W 11/2	14.3	Independence M11-12	p.1019 5-10,13-16, 19-26,33-38
F 11/4	14.3	Independence	worksheet #6
M 11/7	14.4	Green's Theorem	p.1029 1-10,13-20
W 11/9	14.4	Green's Theorem	p.1030 21-24,33-36
F 11/11	Mastery testing 1-14		
M 11/14	14.5	Curl and Div	p.1038 5-10,15-22,27-30
W 11/16	14.6	Surface Integrals	worksheet #7
F 11/18	14.6	Surface Integrals	p.1049 1-6,9-12,21-24,33-38
M 11/21	14.7	Divergence M15-16	p.1060 3-10,15-22,23-28
Thanksgiving			
M 11/28	14.7	continued	worksheet #8
W 11/30	14.8	Stokes Theorem	p.1068 3-10,13-18
F 12/2	14.8	Stokes M17-18	p.1068 21-24
M 12/5	14.9	Applications	p.1075 3-12
W 12/7	Mastery testing 1-19		
F 12/9	Review		
F 12/16	EXAM 8:30-11:30		

Model Reflection Paper

(This is made up, but shows what I'd like to get from you. The two main elements are (1) brief summary of talk and (2) some original thought on the subject.)

The talk on September 7th was by Dr. Sue Dokoo of Pseudo Duke University. Her research is in the game of Sudoku and discussed different aspects of this game. I have seen other people playing it, but did not know the rules or any of the mathematics behind it.

In this game, a 9x9 playing space is provided. An example given was:

		6	2			5	8	
4		2	5					7
				7	8	6		3
5		1		6	7			8
	3						6	
6			8	2		9		1
7		4	3	9				
9					5	2		6
	5	3			1	4		

To "solve" the puzzle, one could just enter numbers in a brute-force kind of way to see if they could get a working configuration. However, sitting in a room full of mathematicians, taking a more analytical approach seemed to be the dominant strategy. Treating this as a constraint-satisfaction problem, you can identify that certain cells must contain specific values. This leads to the conclusion that there is exactly one solution to a "well-formed" Sudoku.

This got me thinking about well-formed Sudoku, and how they are generated in the first place. It seems unlikely that the seeds are randomly assigned, you run the risk of violating set-up rules. A bigger problem is that the seeds may not constrain the possibilities enough to make a unique solution. Another naïve approach might be to take a completed grid and start taking away numbers, but I suspect that you might have a similar issue in terms of necessary constraints.

One that I want to think about is: In forming a viable Sudoku, is it the number of seeds or the placement of seeds that is more critical? I suspect the latter. Also,

- What is the maximum number of seed numbers that can be provided and still result in an ambiguous (unsolvable) puzzle?
- What is the minimum number of seed numbers that can be provided to generate a (uniquely) solvable puzzle?

We were provided two puzzles – one was rated "Easy" the other "Difficult".

- What goes into the rating system?
- Does a difficult puzzle necessarily have fewer seed numbers?
- Is the rating of the complexity somehow determined by the deductive skills required?