**Math 321: Vector Calculus** **2019**

***Calculus***, Smith & Minton 4th edition, Chapters 12-14

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**Course Objectives**: *Continue to learn how to do mathematics!* Mathematics is a problem-solving discipline, and we all have room to improve. To develop as problem-solvers, we must focus on technique and not on memorization. In this course, we develop an understanding of the theory and elementary applications of functions of several variables, multiple integration, line integrals, and integral theorems of vector calculus. Living in a 3-spatial-dimensional world, it is clearly necessary to use such functions if we are to realistically model the world. Unfortunately, the graphical cues that are so helpful for functions of one variable are not as easy to visualize in 3 or more dimensions. The calculations are sometimes more difficult, requiring us to extend our notion of integration. The different types of integrals are beautifully connected at the end of the course in a series of fundamental theorems.

**Intended Learning Outcomes:** At the end of the course, successful students will be able to

- Apply techniques of differentiation and integration to solve problems involving functions of two or more variables and vector functions
- Understand the role of vector calculus in modern mathematics
- Determine when different coordinate systems are appropriate
- Distinguish among various types of integrals, and determine when to use each type

**Attendance Policy**: Attendance and attention in class is essential! Work hard in class and you will have little need for studying the night before a test. You are responsible for everything done in class, through attendance and sharing class notes with classmates. If you miss a class, e-mail or call me before class is over and explain why. With the first unexplained absence, you and your advisor and the registrar will be warned that another unexplained absence will result in removal from the course. **If you have two unexplained absences, you will be dropped from the course.**

**Equipment**: We will use Mathematica in class, on homework assignments and on tests. You are encouraged to get a copy installed on a laptop or desktop. This is free! Take advantage of this great offer. We will practice using Mathematica in class. There will be homework problems and test questions that will be very hard to work without Mathematica! There will be specific homework assignments to get practice using Mathematica. Please note that Mathematica is used extensively in higher-level mathematics courses.

**Study Problems:** In each section of the book that we cover, I will give you a list of problems to study. Work as many of these problems as you can! There are some topics in the book that you are not responsible for learning; the study problems indicate what topics you will see on tests.

**Reports:** A small number of book exercises will be listed for reports to complete. Your best 5 grades *in each half* of the semester count. You may work these by yourself but I encourage you to work in pairs; however, do not repeat partners. Get to know your colleagues and learn from each other!

**Co-Curricular**: During the course of the semester, you must attend at least three approved co-curricular events offered by the MCSP department. For each, write a two-paragraph reflection paper, giving a brief summary of the talk and expanding on some aspect of particular interest to you. Reports are due within a week of the talk. One report must be turned in before fall break.

**I expect you to spend at least 12 hours of work each week inside and outside of class.**

**Academic Integrity**: The college policy is fully supported. Tests are closed notes, closed book unless noted. Electronic devices other than calculators are not allowed in test situations.

**Tests**: Mastery-based testing on the topics on the following page. Test questions will be graded on a mastery/no mastery basis. You will have multiple chances to demonstrate mastery of a topic. Once you do so, you will not be further tested on that topic. So, if you do not achieve mastery of a topic on one test, new problems on that topic will be given on subsequent tests. My judgment of mastery will be based on you demonstrating that you fully understand the question; your arithmetic and algebra do not have to be perfect, but there should be no flaws in your approach to the problem. In some cases, a topic may have multiple parts. If you get part (a) right but not part (b), you will get credit for ½ mastery. When you retake the topic, you must work all parts of the topic. Your grade will not be lowered if you do not master as many parts. While Mastery grading has the downside of no partial credit, the upsides include the ability to completely make up for early deficiencies. You always have the possibility of demonstrating mastery on 100% of the topics for the semester. The intent is not to allow you to delay learning topics to the end of the course, but to encourage you to fully understand the early topics so that later topics can make sense. The exam is Tuesday, December 10, 2:00-5:00 and will consist of all 20 content areas.

**Daily Work**: Each day will start with a problem to work. It is due at 2:25, so be ready to work when class starts. Daily problems may repeat material covered in class the previous time, might review mathematics that will be needed in the day's activities, and might preview the material to be done that day.

**Extra Credit:** You may earn extra credit in a number of ways. My intent is to encourage you to have fun with mathematics, and that is the grading criterion that I will use – so have fun learning! You may check out from the Roanoke College library and report on "popular" mathematics books. You may report on mathematical web sites that have good calculus demonstrations or extra material. You may do one of the extra credit options posted on Inquire during the semester. The main rule here is to do this now; waiting until the end of the semester will distract you from the end-of-semester studying that you need to do.

**Grading**:
Reports: 30% Daily work and co-curricular: 10%
Tests and Exam: 60%
Grades may be curved up based on extenuating circumstances, including improvement as the semester goes on.

| | | |
|---|---|---|
| A: 93-100 | A-: 90-92 | |
| B+: 87-89 | B : 83-86 | B-: 80-82 |
| C+: 77-79 | C: 73-76 | C-: 70-72 |
| D+: 67-69 | D: 63-67 | D-: 60-62 |
| F: 59 and below | | |

**Community:** Welcome to the mathematics community! I hope you will take advantage of the opportunity to get to know me and the outstanding faculty in our department. You may get to know us and other students in the MCSP Study Room (Trexler 271) which is near most of the faculty offices. There is an active Math Club and there is a weekly department tea, which is an informal time to chat, play games, and eat cookies (and drink tea, if you like). Ask me for more information on these and other opportunities such as Stat Crew that I help run.

**Content Areas**

1. Surfaces in Space
2. Parametric Surfaces
3. Gradients
4. Extrema of Functions

5. Double Integrals
6. Double Integrals
7. Area, Volume, Center of Mass
8. Double Integrals in Polar

9. Triple Integrals
10. Cylindrical Coordinates
11. Spherical Coordinates

12. Vector Fields
13. Line Integrals
14. Independence of Path

15. Green's Theorem
16. Curl & Divergence
17. Surface Integrals

18. Divergence Theorem
19. Stokes Theorem
20. Applications

**Grading Scale**

| | |
|---|---|
| **20/20 = 100** | **14/20 = 76** |
| **19/20 = 96** | **13/20 = 72** |
| **18/20 = 92** | **12/20 = 68** |
| **17/20 = 88** | **11/20 = 64** |
| **16/20 = 84** | **10/20 = 60** |
| **15/20 = 80** | **9/20 = 56** |

## Attitude Quiz

Don't be afraid to fail! If you haven't failed frequently, you aren't trying very hard. Another way to look at it is that a missed problem isn't really a failure, it just means that you need more time to try again. Research shows that many of the techniques we learn the best are the ones that we struggle with at first. Give honest answers to the following and think about how you want to approach this course and others at Roanoke College.

**Prep**: Rate yourself 3 (I come to class with questions) to 0 (I don't prepare for class)

**Class**: Rate yourself 3 (I always learn in class) to 0 (I just try to stay awake in class)

**Resourceful**: Rate yourself 3 (I ask professor, others for help) to 0 (I give up easily)

**Relentless**: Rate yourself 3 (I get stuck often) to 0 (I wait to see problems done in class)

**Positive**: Rate yourself 3 (being confused is okay) to 0 (I don't try if it looks hard)

**Tough**: Rate yourself 3 (I try problems every day) to 0 (I only do work the night before)

**Complete**: Rate yourself 3 (I think about whether my answers make sense) to 0 (I just want to get the right answer)

15 and above is a good score. 7 or below and you're not giving yourself much of a chance to succeed (although it may be comfortable because you're not trying hard enough to fail very often, either).

From Jessica Lahey (in the book *How Humans Learn* by Joshua Eyler): "A fear of failure can poison learning by creating aversions to the kinds of experimentation and risk taking that characterize striving."

**Math 321 Schedule**

| Date | Sections | Topics | Study Problems |
|---|---|---|---|
| **W 8/28** | **10.6** | **Introduction** | **p.751 1-2,15,23,24,35,37-38** |
| **F 8/30** | **10.6** | **Quadric Surfaces** | **p.751 3-16,25-27,30-34** |
| **M 9/2** | **11.6** | **Parametric Surfaces** | **p.810 1-5,13-16(M),21-24,27** |
| **W 9/4** | **12.3** | **Partial Derivatives** | **p.849 1-6,11-14,27-28,39-40** |
| **F 9/6** | **12.6** | **Gradient** | **p.879 5-14,19-22** |
| **M 9/9** | **12.6** | **Gradient** | **p.880 27-30,35-36,51-52,57-58** |
| **W 9/11** | **12.7** | **Extrema** | **p.892 1-8,11-14,45-50** |
| **F 9/13** | **TEST** | | |
| **M 9/16** | **13.1** | **Double Integrals** | **p.922 1-2,5-8,11-16,25-30,53-54** |
| **W 9/18** | **13.1** | **Double Integrals** | **p.923 37-46,49-50** |
| **F 9/20** | **13.2** | **Area and Volume** | **p.931 13-18,27-34** |
| **M 9/23** | **13.3** | **Polar Doubles** | **p.939 1-4,7-10,17-24** |
| **W 9/25** | **13.3** | **Polar Doubles** | **p.940 25-28,31-36** |
| **F 9/27** | **TEST** | | |
| **M 9/30** | **13.5** | **Triple Integrals** | **p.954 3-10,13-16,23-28** |
| **W 10/2** | **13.5** | **Triple Integrals** | **p.955 31-38** |
| **F 10/4** | **13.6** | **Cylindrical** | **p.962 9-16,25-32,41-44** |
| **M 10/7** | **13.7** | **Spherical** | **p.969 15-24,27-32,37-40** |
| **W 10/9** | **13.7** | **Spherical** | **p.970 49-54** |
| **F 10/11** | **TEST** | | |

**FALL BREAK**

**Math 321 Schedule**

| Date | Sections | Topics | Study Problems |
|---|---|---|---|
| M 10/21 | 14.1 | Vector Fields | p.995 1-4,9-12,41-46 |
| W 10/23 | 14.1 | Vector Fields | p.996 19-22,29-36 |
| F 10/25 | 14.2 | Line Integrals | p.1010 5-12,17-26 |
| M 10/28 | 14.2 | Line Integrals | p.1010 31-42 |
| W 10/30 | 14.3 | Independence | p.1019 7-10,13-16,19-26 |
| F 11/1 | TEST | | |
| M 11/4 | 14.4 | Green's Theorem | p.1029 1-10,13-20 |
| W 11/6 | 14.4 | Green's Theorem | p.1030 21-24,33-36 |
| F 11/8 | 14.5 | Curl and Div | p.1038 5-10,15-22,27-30 |
| M 11/11 | 14.6 | Surface Integrals | p.1049 1-6,9-12,21-24,33-38 |
| W 11/13 | 14.6 | Surface Integrals | p.1051 45-50,55-58 |
| F 11/15 | TEST | | |
| M 11/18 | 14.7 | Divergence Thm | p.1059 3-10,15-22 |
| W 11/20 | 14.7 | Divergence Thm | p.1060 23-28 |
| F 11/22 | 14.8 | Stokes Theorem | p.1068 3-10,13-18 |
| M 11/25 | 14.8 | Stokes Theorem | p.1068 21-24 |
| Thanksgiving Week | | | |
| M 12/2 | 14.9 | Applications | p.1075 3-12 |
| W 12/4 | TEST | | |
| F 12/6 | Review | | |
| T 12/10 | EXAM 2:00-5:00 | | |

# Math 321 Reports

**Instructions:** During the course of each half-semester (before fall break, after fall break), work at least 5 of the following problems (note that problems are due at fall break and the last day of classes). You may turn in revisions of reports as long as the deadline has not been reached, so it pays to get started early. Your best five grades in each half count. My hope is that you will have fun with these problems. As such, your reports should be complete stories, with introduction, complete work with explanations, and a conclusion. You may earn extra credit by asking a good follow-up question to the problem(s), and more extra credit if you can answer your question! You may turn in hard copy or electronic files (email is fine) but make it easy to read – it is frustrating to try to read a Word file and a Mathematica file at the same time.

**First Half – due date Friday, October 11**

p.751 #42
p.811 #30
p.851 #64
p.881 #60
p.881 #64
p.894 #56
p.923 #58(a)
p.932 #40
p.940 E2
p.955 #54
p.962 #24
p.970 #58

**Second Half – due date Friday, December 6**

p.997 #60
p.1011 #56
p.1020 #40
p.1021 #50
p.1030 #38-40
p.1039 E1
p.1051 #66
p.1060 #36(a)
p.1069 #32
p.1075 #14

## Model Reflection Paper

(This is made up, but shows what I'd like to get from you. The two main elements are (1) brief summary of talk and (2) some original thought on the subject.)

The talk on September 7th was by Dr. Sue Dokoo of Pseudo Duke University. Her research is in the game of Sudoku and discussed different aspects of this game. I have seen other people playing it, but did not know the rules or any of the mathematics behind it.

In this game, a 9x9 playing space is provided. An example given was:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 6 | 2 | | | 5 | 8 | |
| 4 | | 2 | 5 | | | | | 7 |
| | | | | 7 | 8 | 6 | | 3 |
| 5 | | 1 | | 6 | 7 | | | 8 |
| | 3 | | | | | | 6 | |
| 6 | | | 8 | 2 | | 9 | | 1 |
| 7 | | 4 | 3 | 9 | | | | |
| 9 | | | | | 5 | 2 | | 6 |
| | 5 | 3 | | | 1 | 4 | | |

To "solve" the puzzle, one could just enter numbers in a brute-force kind of way to see if they could get a working configuration. However, sitting in a room full of mathematicians, taking a more analytical approach seemed to be the dominant strategy. Treating this as a constraint-satisfaction problem, you can identify that certain cells must contain specific values. This leads to the conclusion that there is exactly one solution to a "well-formed" Sudoku.

This got me thinking about well-formed Sudoku, and how they are generated in the first place. It seems unlikely that the seeds are randomly assigned, you run the risk of violating set-up rules. A bigger problem is that the seeds may not constrain the possibilities enough to make a unique solution. Another naïve approach might be to take a completed grid and start taking away numbers, but I suspect that you might have a similar issue in terms of necessary constraints.

One that I want to think about is: In forming a viable Sudoku, is it the number of seeds or the placement of seeds that is more critical? I suspect the latter. Also,

- What is the maximum number of seed numbers that can be provided and still result in an ambiguous (unsolvable) puzzle?
- What is the minimum number of seed numbers that can be provided to generate a (uniquely) solvable puzzle?

We were provided two puzzles – one was rated "Easy" the other "Difficult".

- What goes into the rating system?
- Does a difficult puzzle necessarily have fewer seed numbers?
- Is the rating of the complexity somehow determined by the deductive skills required?

# Math 321 Information Sheet

Name:

Email:

Cell phone:

Major:

Hometown:

List the math/stat course(s) you took last year.

How hard do you expect this course to be? How interesting?

Why is calculus important?

What are some of the co-curricular or other campus activities you would like to participate in this year?