MATH 381 A, Spring 2017: Real Analysis

Instructor

Dr. David Taylor
Trexler Hall 270B
Email: taylor@roanoke.edu
@RCMathProf

Phone: (540) 375-4933
Fax: (540) 375-2561
Web: see Inquire (inquire.roanoke.edu)
www.facebook.com/uvadt79

Class Meetings

Mondays, Wednesdays, Fridays: 1:10 PM - 2:10 PM in Lucas 125

Office Hours

Mondays, Wednesdays, Fridays: 9:00 AM - 10:30 AM
Tuesdays: 9:00 AM - 10:00 AM, 2:45 PM - 3:30 PM
Thursdays: 10:00 AM - 11:30 AM, 2:45 PM - 3:30 PM and by appointment (email me; take advantage of this, as because of chairperson duties, regularly scheduled office hours may occasionally be shortened or canceled)

Course Information

Real Analysis is one of our upper-level theoretical courses that takes a proof-heavy approach to calculus. Many of the underlying ideas and concepts in this class you have seen before; in this course, we take a rigorous approach to understanding exactly *why* calculus works the way that it does. You may remember a brief coverage of the $\varepsilon - \delta$ definition of a limit discussed in MATH 121; the idea that given any ε we can find a δ in terms of that ε to show that limits exist will be paramount in this course, along with variants of such arguments. Be sure to stay current with material in this course as much of the latter part depends *heavily* on the definitions, theorems, and results established earlier in class.

Intended Learning Outcomes

By the end of this course, successful students will be able to:

- demonstrate an understanding of mathematical language and techniques of mathematical proof.
- understand the concept of continuity and prove standard results about continuous functions.
- understand the concept of differentiability of real-valued functions and prove standard results about differentiable real functions.

Required Materials

Textbook: *Real Analysis for the Undergraduate*; Matthew Pons; Springer-Verlag; ISBN 978-14614-9637-3
Writing Materials: All submitted work for this class should be typed or legible and done in pen or pencil; do not use blue ink for your submitted work.

Course Grades

The following table lists the weights for the various forms of assessment for this class.

Daily Homework Problems	15%
Weekly Homework Sets	15%
Definition/Theorem Quizzes	10%
Tests (20% each)	40%
Final Exam	20%

A grade scale will be determined after final grades are computed, but will be no worse than the scale given below. Attendance and class participation will be considered when determining marginal grades.

		B+	87-89	C+	77-79	D+	67-69		
A	93-100	B	83-86	C	73-76	D	63-66	F	0-59
A-	90-92	B-	80-82	C-	70-72	D-	60-62		

Participation

The key to learning a topic in mathematics is participation. We will strive to have an active, rather than passive, classroom environment. This means that questions are welcome in class, and the instructor will ask for your help in completing problems and proofs presented in class. Depending on your learning style, you may prefer to see a topic in class, and then read the textbook, or perhaps read the textbook before class and then see it explained further in class; either way you prefer, *reading the textbook* and paying close attention will be key in your success.

Homework

There will be two types of homework assignments given in class. Each day, the instructor will assign one problem that will be due at the start of the next class; these daily homework problems are designed to keep you current with the material and also allow you more frequent feedback about your progress in class. *Please start these problems early, as it may take a few hours to fully understand and start thinking about what the problem is asking.*

On a weekly basis, larger problem sets will be assigned, usually consisting of about 4 problems. These problem sets are designed so that you can more fully engage the material that we are learning about in class. Once assigned, you will have approximately one week to complete these. Again, *please start working on these problems early* as it will take time to completely digest the material.

Unless otherwise specified, you are encouraged to talk to other students in the class while working on your homework. You should, however, *write up your solutions entirely by yourself and on your own*. Mathematics is meant to be discussed, but the final write-up must be done on your own and in your own words.

Quizzes

There will be quizzes in this class, at least on a weekly basis. These quizzes will ask you to "regurgitate" some of the definitions, theorems, and results that we see in class and in the textbook. Learning mathematics is often like learning a language; knowing the vocabulary and results is a prerequisite for completing the proofs and problems in this class. Generally speaking, you should have a good idea beforehand of which definitions and such will appear, but you are responsible for *all* definitions, theorems, and results that appear in the textbook for sections that we cover. At most one problem on a quiz may be reserved for a definition, theorem, or result that appears in a *previously covered* part of our class.

Note that you do not need to memorize anything word for word; what you indicate as your definition, theorem, or result must, however, be mathematically equivalent to the instructor's or textbook's version of the question.

Tests and Final

There will be two midterm tests in this class along with a final exam. You should think of the final exam as a third test; each of the three tests will cover a specific part of the textbook. While these tests are not designed to be cumulative, because of the cumulative nature of mathematics and its progression of material, mastery of previous material will be beneficial to success on future tests. Please take time to learn from your homework feedback and previous test feedback throughout this class. The instructor is always more than happy to discuss feedback in more detail.

Each test will be done in the take-home format and you will have approximately one week to complete each test. Note that, as with homework, *starting early* will be key to doing well on these tests! Understanding the questions and allowing yourself time to process your thoughts over a few days is an excellent strategy to having solutions find themselves to you. Starting early also gives you time to discuss any clarifying points with the instructor so that you still have time to devote to the problems.

Writing Proofs and Grading

One of the primary goals for this course is further development of your logical reasoning and proof writing skills. A proof should start off by assuming the hypotheses given and end by a statement of what was to be proved. Your job is to fill in this middle part by providing a prose paragraph (or more) that uses what you have learned to move from fact to fact to reach the end.

The final result should be polished and presented in mostly prose format with justification provided for each step that you provide. The use of shorthand symbols is extremely uncommon in polished and published proofs (for instance, I have never once seen the symbols for "therefore" and "such as" used in published mathematical proofs); when in doubt, it's best to words over symbols. The main exception is when defining sets, as in

$$X = \{x \in \mathrm{R} | \forall y \in C, x \geq y\}$$

which defines the set X to be the set of all real numbers x that happen to be less than all values y in an already defined set C. Here, the symbolic use is typically set aside as above. In a true proof, this "offset" statement would be proceeded by the word "Let" and followed by more English prose to complete the sentence.

Feedback given on your proofs will be both mathematical in nature and stylistic in nature so that you can improve as the semester goes on. Note, however, that *grades will only be given and based upon the mathematical correctness of your work*. Stylistic suggestions for improvement will not affect your grade in any way.

MCSP Conversations

As you are aware, the MCSP Department offers a series of discussions that appeal to a broad range of interests related to these fields of study. These co-curricular sessions will engage the community to think about ongoing research, novel applications and other issues that face our discipline. You are invited be involved with all of these meetings; however participation in at least three of these sessions is mandatory. After attending, students will submit a one page paper reflecting on the discussion. This should **not** simply be a regurgitation of the content, but rather a personal contemplation of the experience. These reaction papers will be counted as a quiz and must be submitted using our Inquire website.

Attendance & Make-Up Work

Attendance is critical to the understanding of the material in the course; it is both required and expected. Any absence that is not discussed with the instructor prior to the missed class is considered unexcused. When absent, excused or unexcused, you are responsible for all material covered in class. You will not be allowed to make up any work missed due to an unexcused absence. Late homework will be accepted without penalty as long as the instructor has not graded the homework; afterwards, late homework will be accepted, up until the start of the next class period, with a 50% penalty (after grading) assessed. Take-home exams *must* be turned in on-time; a late-penalty (after grading) of $33\frac{1}{3}\%$ will be assessed per 24 hours late (rounded up); *no exceptions*.

Study Room

The MCSP Study Room, Trexler 271, can be used by you and your friends to meet up so that you can work on homework together or prepare for tests. It is open virtually 24 hours a day, 7 days a week (very occasionally there are meetings in that room). Your student ID card should grant you access to Trexler Hall any time of day if the doors happen to be locked (use the card access point located by the first floor entrance facing the parking lot). Take advantage of this area and time, especially during weekdays when I am around (which is generally a lot)!

Community

Please feel free to become an active member of our department's community. Each of the three disciplines in our department has a student club and you should join! The Roanoke College Student Chapter of the Mathematical Association of America (or "Math Club" for short) meetings every other week, plays and learns about games and hosts evening events and the annual Pi-Day celebration! Membership in our Math Club also grants membership into the MAA itself; one of the premiere professional mathematical organizations in the world.

In addition, our department offers MCSP Tea every week so come by Trexler 271 to talk to and meet other students as well as chat with the MCSP faculty members in a casual setting! The time is yet to be determined and will be posted on flyers around Trexler Hall and also advertised in class.

Academic Integrity

Students are expected to adhere to the Academic Integrity policies of Roanoke College. All work submitted for a grade is to be your own work! No electronic devices can be used during any class or testing period (this includes cell phones; please silence or turn them **off** before class). Note that looking at or using your cell phone during a quiz is considered a violation of Academic Integrity regardless of your purpose or intent in doing so.

Course Schedule

The pace of this class will more or less be dictated by the class itself, so a day-by-day schedule is not provided on this syllabus. Instead here is a list of the textbook sections we aim to cover at a minimum, along with dates that take home exams will be given and due.

Sections to Cover	
1.1 Preliminaries	3.3 Absolute and Conditional Convergence
1.2 Complete Ordered Fields	4.1 Sequences and the Limit of a Function
1.3 The Real Number System	4.2 Continuity
1.4 Set Structures in R	4.3 The Intermediate Value Theorem
2.1 Sequences and Convergence	4.4 Continuity of a Set and Uniform Continuity
2.2 Properties of Convergent Sequences	5.1 The Definition of the Derivative
2.3 Completeness in R Revisited	5.2 Properties of the Derivative
2.4 Set Structures in R via Sequences	5.3 Value Theorems for the Derivative
3.1 Series of Real Numbers	5.4 Consequences of the Value Theorem
3.2 Basic Convergence Tests	5.5 Taylor Polynomials

Test Schedule	
Test 1	Given Out on Wednesday, February 15 Due on **Wednesday, February 22** by 3:00 PM
Test 2	Given Out on Friday, March 24 Due on **Friday, March 31** by 3:00 PM
Final Exam	Given Out on Monday, April 24 Due on **Monday, May 1** by 5:00 PM

Course Work Load

At a bare minimum, this course expects you to spend at least 12 hours of work each week inside and outside of class.