Course Objectives: Continue to learn mathematics! Discrete mathematics is an introduction to proofs. Proofs, in turn, reflect the critical thinking skills, precision and rigor that characterize mathematics and programming. Logical deduction is critical in any number of disciplines: a Roanoke grad who finished second in his class at Duke Law School credits discrete mathematics as the most important undergraduate course he took. The main objectives of this course are to improve your thinking skills to enhance your success in whatever your profession becomes, and to enjoy the problem-solving process.

Intended Learning Outcomes: At the end of the course, successful students will be able to

- Describe and apply each of several methods of mathematical proof
- Analyze a logical argument, and identify and correct any logical fallacies
- State and prove fundamental facts in number theory and graph theory
- Apply recursive relations to construct and prove facts about sequences

Equipment: This is a thinking course. We will not use Mathematica or other software to any significant degree. It is always a good idea to take notes. The act of writing down an idea gives you a muscle memory that helps your recall of the idea.

Attendance Policy: Regular attendance is expected. You must keep up with definitions! You are responsible for everything done in class. If you miss a class, e-mail or call me before class is over and explain why. If you have two unexplained absences, you may be dropped from the course after a warning email is sent.

## I expect you to spend at least 12 hours of work each week inside and outside of class.

Academic Integrity: The college policy is fully supported. Tests and are closed notes, closed book. Homework assignments will be discussed in class, and you may always ask me for help. Do not collaborate on homework. No electronic devices are allowed in a test situation.

Study problems and homework: Problems from each section of the book will be suggested. These are not to be turned in, but you should work each problem and ask questions about the ones you have trouble with. Study problems indicate the types of problems you will see on tests. In addition, you will be asked to turn in designated problems that go beyond the basic problems in the book. You will be given several days to work on these, which will be graded on the quality of the writing as well as the mathematics. In general, homework problems will not be directly included on tests.

Daily problems: On most days, a short problem or puzzle will be projected for you to work at the beginning of class. These will be turned in by $1: 15$ so come to class on time and ready to work!

Tests: There will be six tests and a final exam. Each test will cover all material discussed since the previous test. Anticipated test dates are every other Friday: 2/2, 2/16, 3/2, 3/23, 4/6, 4/20. The exam is Monday, April 30, 2:00-5:00.

Make-ups: In case of sickness or scheduling conflicts, get in touch with me ASAP.

Co-Curricular: During the course of the semester, you must attend at least two approved cocurricular events offered by the MCSP department. For each, write a two-paragraph description of the event, due within a week of the event. A sample is provided. Schedules for the talks can be found online at the MCSP department website and in Trexler hallways. http://www.roanoke.edu/inside/a-z index/math cs and physics/conversation series/spring 2018

Grading: Homework counts $20 \%$ (two tests!). The final counts $10 \%$. Each test counts $10 \%$ (for $60 \%$ total) of the final average. Tests are graded on a point scale, not with mastery grading. Grades may be curved up based on participation, one unusually low test score or some other extenuating circumstance.
A: 93-100 A-: 90-92
B+: 87-89 B : 83-86 B-: 80-82
C+: 77-79 C: 73-76 C-: 70-72
D+: 67-69 D: 63-67 D-: 60-62 F: 59 and below
Special Needs: located in the Goode-Pasfield Center for Learning and Teaching in Fintel Library, provides reasonable accommodations to students with identified disabilities. Reasonable accommodations are provided based on the diagnosed disability and the recommendations of the professional evaluator. In order to be considered for disability services, students must identify themselves to the Office of Accessible Education Services. Students requesting accommodations are required to provide specific current documentation of their disabilities. Please contact Accessible Education Services, at 540-375-2247.
If you are on record with Accessible Education Services as having academic or physical needs requiring accommodations, please schedule an appointment with Accessible Education Services as soon as possible. You need to discuss your accommodations before they can be implemented. Also, please note that arrangements for extended time on exams, testing, and quizzes in a distraction-reduced environment must be made at least one week before every exam.

Extra Credit: You may earn extra credit in a number of ways. My intent is to encourage you to have fun with mathematics, and that is the grading criterion that I will use - so have fun learning! You may check out from the Roanoke College library and report on "popular" mathematics books. You may report on mathematical web sites that have good mathematics demonstrations or extra material. You may do one of the extra credit options posted on Inquire during the semester. The main rule here is to do this now; waiting until the end of the semester will make it hard to get much credit and (more importantly) distract you from the end-of-semester studying that you need to do.

| Date | Topic | Section | Study |
| :---: | :---: | :---: | :---: |
| 1/15 | Introduction |  |  |
| 1/17 | Language of Sets | 1.2,1.3 | p13 \#1,3,4,7,8,11; p21 \#1,5,9,11,12 |
| 1/19 | Logical Form | 2.1 | p36 \#5,6,8,11,13,14,17,21,23,26,28,40 |
|  | HW \#1 DUE |  |  |
| 1/22 | Conditionals | 2.2 | p48 \# 5,7,16,17,20,24,28,40-43 |
| 1/24 | Arguments | 2.3 | p60 \#1,2,6,8,25-29,36 |
| 1/26 | Quantified | 3.1 | p73 \#5,7,9,10,13,16,23,25,28 |
|  | HW \#2 DUE |  |  |
| 1/29 | Quantified II | 3.2 | p83 \#2-4,7,9,10,13,20-23,34,38,39 |
| 1/31 | Multiple Qs | 3.3 | p93 \#9,11,12,15,16,29,31,35,36,41 |
| 2/2 | TEST \#1 |  | 1.2-3.2 |
| 2/5 | Arguments | 3.4 | p106 \#8-11,15-17,19-23,31,33 |
| 2/7 | Direct Proof | 4.1 | p125 \#5,6,11-12,14,21-23 |


| Date | Topic | Section | Study |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| $2 / 9$ | Direct Proof | 4.1 | p 126 \#27-32,43-46,50,52,57-60 |
|  | HW \#3 DUE |  |  |
| $2 / 12$ | Rationals | 4.2 | p 132 \#14-20,28,30,35-38 |
|  |  |  |  |
| $2 / 14$ | Divisibility | 4.3 | p 142 \#9,10,13,15,24-30,33,36 |
|  |  |  |  |
| $2 / 16$ | TEST \#2 |  | $3.3-4.2$ |
|  |  | 4.5 | p 162 \#7-13,19-21,28-30 |
| $2 / 19$ | Quotient-Remainder | 4.4 | p 153 \#7,8,13,17,19,20,23,24,28,30,36-38 |
|  |  | 4.6 | $\mathrm{p} 169 \# 7-12,19,23$ |
| $2 / 21$ | Contradiction |  |  |
|  |  | 5.1 | p 184 \#8-11,19-23,32,36,38-40,48-51, |
| $2 / 23$ | Two Classics | HW \#4 DUE | 5.2 |
|  | pequences |  |  |
| $2 / 26$ |  |  | $4.3-5.1$ |
|  | Induction I |  |  |
| $2 / 28$ |  | TEST \#3 |  |
| $3 / 2$ |  |  |  |
|  | SPRING BREAK |  |  |
|  |  |  |  |


| Date | Topic | Section | Study |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| $3 / 12$ | Induction II | 5.3 | p 207 \#3-4,13-17,24-26,36 |
|  |  |  |  |
| $3 / 14$ | Strong Induction | 5.4 | p 219 \#3-8,10,18-19,25 |
|  |  |  |  |
| $3 / 16$ | Recursion | 5.5 | p 234 \#1-4,9-11,25-33,36-37 |
|  | HW \#5 DUE |  |  |
| $3 / 19$ | Solving Relations | 5.6 | p 247 \#5-10,18-19,25-26,45-48 |
|  |  | 6.1 | p 262 \#6-10,13,17-20,24,27-28,31,34 |
| $3 / 21$ | Set Theory |  |  |
|  |  |  | $5.2-5.6$ |
| $3 / 23$ | TEST \#4 | 6.2 | p 277 \#7-12,15-21,36 |
|  |  |  |  |
| $3 / 26$ | Set Properties | 6.3 | p 284 \#7-12,17-18,30-34,44 |
|  |  |  |  |
| $3 / 28$ | Algebraic Proofs | HW \#6 DUE |  |
|  | GOOD FRIDAY |  |  |
| $3 / 30$ |  |  |  |
|  |  |  |  |


| Date | Topic | Section | Study |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| $4 / 2$ | Functions | 7.1 | p 302 \#1-3,7-8,11-16,31-36 |
|  |  |  |  |
| $4 / 4$ | One-to-One | 7.2 | p 319 \#1-2,5-7,9-10,14-18,22-23 |
|  |  |  |  |
| $4 / 6$ | TEST \#5 | 7.4 | p 343 \#1-5,9-13,25,29-30,32 |
|  |  | 8.4 | p 386 \#9-10,12,14,20-21 |
| $4 / 9$ | Cardinality | 9.4 | p 444 \#1-4,7,10,20-22,25-27,33 |
|  |  |  |  |
| $4 / 11$ | Mod Arithmetic | 9.6 | p 473 \#1-4,10-12,17-23,29-32,43-44 |
|  |  |  |  |
| $4 / 13$ | Pigeonholes |  |  |
|  | HW \#7 DUE |  |  |
| $4 / 16$ | Pascal's Formula |  | $7.2-9.6$ |
|  |  |  |  |
| $4 / 18$ | Review for Test 6 |  |  |
|  |  |  |  |
| $4 / 20$ | TEST \#6 |  |  |
|  |  |  |  |
| $4 / 23$ | Review for Exam |  |  |
|  |  |  |  |
| $4 / 30$ | FINAL EXAM |  |  |
|  |  |  |  |

## Math 131 Homework \#1

This is due on Friday, January 19, at the beginning of class. Late papers lose $15 \%$ per day or partial day late. The product of this assignment is a report, with clear explanations (which may be in mathematical language rather than English). Type your report and submit it electronically - by email, not Inquire.

Work the following two logic puzzles. In each case, include the question, your answer, and a full explanation of your result.

FYI: These are examples of puzzles that smart people often get wrong when they "think fast" but get right when they "think slow" (terms used by author Daniel Kahneman in the excellent book Thinking Fast and Slow). This course is all about developing your "slow" thinking skills.

1. There are four cards, each of which has a letter on one side and a number on the other. The company printing the cards was given the instruction
"If there is a vowel on one side, there must be an odd number on the other side."
The four cards are placed on a table with the following sides up:
E G 85
Which cards must be turned over to determine if the printer followed the instruction correctly?
2. Linda is 31, single, outspoken, and socially conscious. Which is more likely,
(a) Linda is a bank teller or
(b) Linda is a bank teller who is active in the feminist movement.

## Model Reflection Paper

(This is made up, but shows what I'd like to get from you. The two main elements are (1) brief summary of talk and (2) some original thought on the subject.)

The talk on September $7^{\text {th }}$ was by Dr. Sue Dokoo of Pseudo Duke University. Her research is in the game of Sudoku and discussed different aspects of this game. I have seen other people playing it, but did not know the rules or any of the mathematics behind it.

In this game, a 9 x 9 playing space is provided. An example given was:

|  |  | 6 | 2 |  |  | 5 | 8 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 4 |  | 2 | 5 |  |  |  |  | 7 |
|  |  |  |  | 7 | 8 | 6 |  | 3 |
| 5 |  | 1 |  | 6 | 7 |  |  | 8 |
|  | 3 |  |  |  |  |  | 6 |  |
| 6 |  |  | 8 | 2 |  | 9 |  | 1 |
| 7 |  | 4 | 3 | 9 |  |  |  |  |
| 9 |  |  |  |  | 5 | 2 |  | 6 |
|  | 5 | 3 |  |  | 1 | 4 |  |  |

To "solve" the puzzle, one could just enter numbers in a brute-force kind of way to see if they could get a working configuration. However, sitting in a room full of mathematicians, taking a more analytical approach seemed to be the dominant strategy. Treating this as a constraintsatisfaction problem, you can identify that certain cells must contain specific values. This leads to the conclusion that there is exactly one solution to a "well-formed" Sudoku.

This got me thinking about well-formed Sudoku, and how they are generated in the first place. It seems unlikely that the seeds are randomly assigned, you run the risk of violating set-up rules. A bigger problem is that the seeds may not constrain the possibilities enough to make a unique solution. Another naïve approach might be to take a completed grid and start taking away numbers, but I suspect that you might have a similar issue in terms of necessary constraints.

One that I want to think about is: In forming a viable Sudoku, is it the number of seeds or the placement of seeds that is more critical? I suspect the latter. Also,

- What is the maximum number of seed numbers that can be provided and still result in an ambiguous (unsolvable) puzzle?
- What is the minimum number of seed numbers that can be provided to generate a (uniquely) solvable puzzle?
We were provided two puzzles - one was rated "Easy" the other "Difficult".
- What goes into the rating system?
- Does a difficult puzzle necessarily have fewer seed numbers?
- Is the rating of the complexity somehow determined by the deductive skills required?


## Math 131 Information Sheet

Name:
Intended Major:
Hometown:
List any other college math courses you have taken.

What are your expectations and goals for this course?

What campus activities do you expect to be involved in this semester?

