

# Mathematical Statistics

STAT 301/ Spring 2019

**Instructor:** Roland Minton / [minton@roanoke.edu](mailto:minton@roanoke.edu)

**Office:** Trexler 270C

**Phone:** 540-375-2358

**Office Hours:** by appointment at [calendly.com/Minton/15min](https://calendly.com/Minton/15min).

**Required Text:** *Mathematical Statistics with Application 7<sup>th</sup> Edition*, by Wackerly, Mendenhall, and Scheaffer.

**Course Objective:** The objective of this course is to expand our understanding of probability theory in order to clearly understand the role that it plays in statistics. We will investigate functions of random variables in order to define point estimators and understand their properties. While developing a strong understanding the theoretical aspects of statistical methods we will concurrently work on applied problems and handle real data sets.

**Intended Learning Outcomes:** By the end of this course, students will be able to ....

- ...understand and interpret multivariate probability distributions.
- ...identify relationships between random variables.
- ...articulate the connections between probability theory and statistic.
- ...describe the role the central limit theorem plays in probability and statistics.
- ...understand the properties of point estimators and their connection with point estimates.

**Content:** We will cover most of chapters 5 through 10 in the text as well as part of chapter 16. Included in these chapters is:

- Multivariate Probability Distributions
- Functions of Random Variables
- Sampling Distributions and the Central Limit Theorem
- Estimators
- Properties of Point Estimators and Methods of Estimation
- Hypothesis Testing
- Bayesian Methods

**Tests:** Mastery-based testing with topics shown below.

**Homework/Assignments:** Homework assignments will be given regularly. Assignments will come from the book as well as projects using Mathematica, and R.

**Final Exam:** The exam is Tuesday April 30, 2:00-5:00.

**Grading:** Homework (including Conversation Series) counts 30% and tests/exams count 70%.

A tentative guideline for determination of grade will then be:

|    |           |    |           |    |           |    |           |
|----|-----------|----|-----------|----|-----------|----|-----------|
| A  | > 93      | B  | 83 – 86.9 | C  | 73 – 76.9 | D  | 63 – 66.9 |
| A- | 90 – 93   | B- | 80 – 82.9 | C- | 70 – 72.9 | D- | 60 – 62.9 |
| B+ | 87 – 89.9 | C+ | 77 – 79.9 | D+ | 67 – 69.9 | F  | < 60      |

**MCSP Conversation Series:** Attending at least two MCSP conversation series events is required. Within one week of the lecture, a one page reflection paper will be due.

**Expected Hours of Work:** This course expects you to spend at least 12 hours of work each week inside and outside of class.

**Attendance:** Attendance is required and expected and is crucial to be successful in this course. An absence that is not discussed with the instructor prior to class is considered unexcused. Regardless of whether the absence is excused or not, you are responsible for all the material covered in class.

**Technology:** Scientific calculators, Mathematica, and R will be used throughout the semester in the classroom and on assignments. Cell phones are expected to be turned off before entering the class and computers will be used in the classroom exclusively for academic purposes.

**Academic Integrity System:** The Roanoke College Academic Integrity System applies to all graded work in this course. Students are responsible for understanding and adhering to the Academic Integrity System. Among other things the Academic Integrity System prohibits giving or receiving unauthorized aid, assistance, or unfair advantage on academic work. Please note that having a phone or unauthorized electronic device out during a test is an academic integrity violation.

**Mastery Topics:**

1. Multinomial Distributions
2. Bivariate Normal Distributions
3. Method of Distribution Functions
4. Method of Transformations
5. Method of Moment-Generating Functions
6. Central Limit Theorem
7. Unbiased Point Estimators
8. Goodness of Fit
9. Confidence Intervals for Means and Proportions
10. Confidence Intervals for Variance
11. Efficiency and Consistency
12. MVUE
13. Method of Percentiles
14. Maximum Likelihood
15. Large Sample Tests
16. Small Sample Tests
17. Tests for Variance
18. Power of Tests
19. Likelihood Ratio Test
20. Chi-Square Test
21. Contingency Tables
22. Bayesian estimators
23. Bayesian credible intervals
24. Information criterion

Grading scale:

Master  $m$  topics, grade is  $28 + 3m$ .

**Daily Schedule** (tentative: may be changed by weather cancellations)

|        |                                                                       |
|--------|-----------------------------------------------------------------------|
| M 1/14 | review of 5.2-5.7 bivariate distribution                              |
| W 1/16 | 5.9 multinomial distributions                                         |
| F 1/18 | 5.10 bivariate normal distribution                                    |
| M 1/21 | 6.2 finding probability distributions                                 |
| W 1/23 | 6.3 method of distribution functions                                  |
| F 1/25 | 6.4 method of transformations                                         |
| M 1/28 | 6.5 method of moment generating functions                             |
| W 1/30 | 7.2 sampling distributions                                            |
| F 2/1  | 7.3 Central Limit Theorem                                             |
| M 2/4  | 7.5 normal approximation to binomial                                  |
| W 2/6  | review                                                                |
| F 2/8  | Test #1                                                               |
| M 2/11 | 8.2 bias and MSE of point estimators                                  |
| W 2/13 | 8.3 common unbiased point estimators                                  |
| F 2/15 | 8.4 evaluating goodness of fit                                        |
| M 2/18 | 8.5-6 large-sample confidence intervals                               |
| W 2/20 | 8.8 small-sample confidence intervals                                 |
| F 2/22 | 8.9 confidence intervals for variance                                 |
| M 2/25 | 9.2 relative efficiency                                               |
| W 2/27 | 9.3 consistency                                                       |
| F 3/1  | 9.5 Rao-Blackwell Theorem and MVUE                                    |
| M 3/11 | 9.6 methods of moments and percentile matching                        |
| W 3/13 | review                                                                |
| F 3/15 | Test #2                                                               |
| M 3/18 | 9.7 method of maximum likelihood                                      |
| W 3/20 | 10.3 common large sample tests                                        |
| F 3/22 | 10.5 relationship between hypothesis testing and confidence intervals |
| M 3/25 | 10.6 attained significance levels                                     |
| W 3/27 | 10.8 small-sample hypothesis testing: $\mu$ , $\mu_1 - \mu_2$ , $p$   |
| F 3/29 | 10.9 testing hypotheses concerning variance                           |
| M 4/1  | 10.10 power of tests and Neyman-Pearson Lemma                         |
| W 4/3  | 10.11 likelihood ratio tests                                          |
| F 4/5  | 14.2 chi-square tests                                                 |
| M 4/8  | review                                                                |
| W 4/10 | Test #3                                                               |
| F 4/12 | 14.4 contingency tables                                               |
| M 4/15 | 16.2 Bayesian priors, posteriors, and estimators                      |
| W 4/17 | 16.3 Bayesian credible intervals                                      |
| F 4/19 | 16.4 Bayesian tests of hypotheses: information criteria               |